Gravitational Potential Energy Worksheet

① $E_g = -\frac{GMm}{r} = \frac{-(6.67\times10^{-11})(5.98\times10^{24})(1)}{6.38\times10^{6}}$

$= -6.25\times10^{7}\ J$

② $E_g = -\frac{GMm}{r} = \frac{-(6.67\times10^{-11})(5.98\times10^{24})(7.34\times10^{22})}{3.8\times10^{8}}$

$= -7.7\times10^{28}\ J$

③ $\Delta E_g = -\frac{GMm}{r_f} - \left(-\frac{GMm}{r_i}\right)$

$= GMm\left(\frac{-1}{r_f} + \frac{1}{r_i}\right)$

$= (6.67\times10^{-11})(5.98\times10^{24})(1)\left(\frac{-1}{2(6.38\times10^{6})} + \frac{1}{6.38\times10^{6}}\right)$

$= 3.13\times10^{7}\ J$

④ $\Delta E_g = -\frac{GMm}{r_f} - \left(-\frac{GMm}{r_i}\right) = GMm\left(-\frac{1}{r_f} + \frac{1}{r_i}\right)$

$= (6.67\times10^{-11})(5.98\times10^{24})(5)\left(\frac{-1}{1.25\times(6.38\times10^{6})} + \frac{1}{6.38\times10^{6}}\right)$

$= 6.25\times10^{7}\ J$

⑤ $K_i + U_i = K_f + U_f$

$$-\frac{GMm}{r_i} = \frac{1}{2}mv^2 - \frac{GMm}{r_f}$$

$$\frac{1}{2}mv^2 = \frac{GMm}{r_f} - \frac{GMm}{r_i}$$

$$v = \sqrt{2GM\left(\frac{1}{r_f} - \frac{1}{r_i}\right)}$$

$$= \sqrt{2(6.67\times10^{-11})(7.34\times10^{22})\left(\frac{1}{1.74\times10^6} - \frac{1}{(1.05)1.74\times10^6}\right)}$$

$$\underline{v = 518 \text{ m/s}}$$

⑥ $K_i + U_i = K_f + U_f$

$$\frac{1}{2}mv^2 - \frac{GMm}{r_i} = -\frac{GMm}{r_f}$$

$$v = \sqrt{2GM\left(\frac{1}{r_i} - \frac{1}{r_f}\right)}$$

$$= \sqrt{2(6.67\times10^{-11})(5.98\times10^{24})\left(\frac{1}{6.38\times10^6} - \frac{1}{2(6.38\times10^6)}\right)}$$

$$\underline{v = 7900 \text{ m/s}}$$

⑦ $\Delta E_g = -\frac{GMm}{r_f} - \left(-\frac{GMm}{r_i}\right)$

$$= GMm\left(\frac{-1}{r_f} + \frac{1}{r_i}\right)$$

$$= 6.67\times10^{-11}(5.98\times10^{24})(1)\left(\frac{-1}{6.38\times10^{6}+100\times10^{3}} + \frac{1}{6.38\times10^{6}}\right)$$

$\Delta E_g = 9.6\times10^{5}$ J

$\Delta E_g = mg\Delta h = 1(9.8)(100\times10^{3}) = 9.8\times10^{5}$ J

$\frac{9.8\times10^{5} - 9.6\times10^{5}}{9.6\times10^{5}} = 2\%$ error (difference)

For most purposes at heights less than or equal to 100 km, the equation mgh is close enough.

8 (a) $\Delta E_g = -\frac{GMm}{r_f} - \left(-\frac{GMm}{r_i}\right)$

$= GMm\left(\frac{-1}{r_f} + \frac{1}{r_i}\right)$

$= 6.67\times10^{-11}(1.98\times10^{30})(5.98\times10^{24})\left(\frac{-1}{1.52\times10^{11}} + \frac{1}{1.47\times10^{11}}\right)$

$\Delta E_g = 1.77\times10^{32}\ J$

(b) The earth is moving fastest when it is closest to the sun (perihelion).

The change in kinetic energy is the same as the change in gravitational potential energy,

Gravitational Potential Energy Worksheet

1. What is the value of the gravitational potential energy of a 1.00 kg mass on the surface of the earth if the zero of potential energy is taken to be at infinity? (-6.25×10^{7} J)

2. What is the gravitational potential energy of the moon with respect to the earth if the zero of potential energy is taken to be at infinity? ($-7.7\times10^{28}\ J$)

3. What is the change in gravitational potential energy of a 1.00 kg mass that is carried from the surface of the earth to a distance of one earth radius above the surface? ($3.13\times10^{7}\ J$)

4. What is the change in gravitational potential energy of a 5.00 kg mass that is carried from the surface of the earth to a distance of 0.25 earth's radius above the surface? ($6.26\times10^{7}\ J$)

5. A metal slug is dropped from a height of $0.05r_m$ above the moon's surface. Find the speed with which the slug strikes the moon's surface. ($518\ m/s$)

6. With what initial velocity must an object be projected vertically upward from the surface of Earth, in order to rise to a height equal to Earth's radius? ($7.9\times10^{3}\ m/s$)

7. Calculate the change in gravitational potential energy for a 1 kg mass lifted 100 km above Earth's surface. What percentage error would have been made by using the equation $E_g = mgh$ and the value of g at Earth's surface? What does this tell you about the need for the more exact treatment in most normal Earth-bound problems? ($1.0\times10^{6}\ J$, 2%)

8. The distance from the sun to Earth varies from $1.47\times10^{11}\ m$, at perihelion (closest approach), to $1.52\times10^{11}\ m$ at aphelion (farthest distance away).
   a. What is the maximum change in Earth's gravitational potential energy during one orbit of the sun? ($1.8\times10^{32}\ J$)
   b. At what point in its orbit is Earth moving the fastest, and what is its maximum change in kinetic energy? (perihelion, $1.8\times10^{32}\ J$)